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# LIFE AGENT'S AID:

An Essay

UPON THE

ORIGIN OF THE SURPLUS

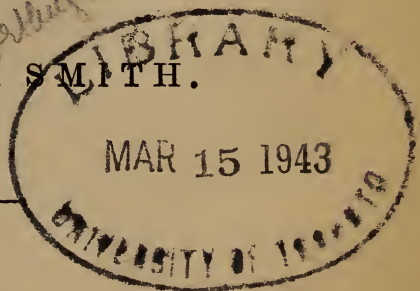
OF

LIFE INSURANCE COMPANIES,

TOGETHER WITH AN

ARITHMETICAL EXPLANATION OF THE CONTRIBUTION  
PLAN OF DISTRIBUTION.

By HENRY W. SMITH.



New York:

INSURANCE MONITOR OFFICE

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## P R E F A C E.

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THE author of this monograph deems it proper to state that the material used has been gathered from various sources, and that the mathematical principles involved have in one shape or another been placed before the public very many times. Nothing on the score of originality is claimed. The merits of the work, if any it has, are due to its compactness and availability.

To increase its usefulness, the more common Monetary Tables, and Tables of Net Annuities and Premiums computed from "The Combined Experience, or Actuary Rates of Mortality," with interest at four per cent., have been added in the Appendix.

In giving this essay to the public, all the writer dares hope is, that it may stimulate the life agent to further investigation. If through its instrumentality he can aid in the diffusion of information concerning Life Contingencies, he will feel that his labors have not been entirely useless.

H. W. S.

February 1st, 1869.



## THE LIFE AGENT'S AID.

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It will be the province of this essay to ascertain the sources from which the surplus of Life Assurance Societies arises; to state the usual methods that have been adopted for its division; and to give, illustrated by arithmetical processes, an intelligible explanation of the Contribution Plan recently adopted by a considerable number of the American Companies. In furtherance of this design we deem it essential to call the attention of the reader to the fundamental principles upon which Mutual Life Assurance Societies are founded.

The science of Life Contingencies is based upon the doctrine of chances. These, paradoxical as it may seem, are subject to a law, the operation of which is as regular as that of gravitation. If we examine the history of any large number of happenings, we shall find that they have recurred with surprising uniformity. For example, the number of aces which come up in throwing a die ten thousand times, indicates with a good degree of accuracy how many will come up in the next ten thousand throws. Extended data, in relation to the number of casualties of any description that have taken place in a given locality, will enable us to predict with tolerable certainty in respect to the recurrence of similar events in the future. This is well illustrated by the number of violent deaths that occur

red in Great Britain during five consecutive years. They were as follows :—

In	1848	1849	1850	1851	1852
	13,551	13,324	13,987	13,559	14,475

Of this number there died from poison :—

In	1848	1849	1850	1851	1852
	569	526	553	528	553

Murders seem to be subject to the same law of average. We give statistics from M. Quetelot, relating to the number of murderers in France brought to justice during the following years :—

In	1826	1827	1828	1829	1830	1831
	241	234	227	231	207	266

While, at the end of the year 1852, it would have been easy to predict the number of deaths by casualty that would have occurred in Great Britain in 1853; or in 1831 to foretell the number of murderers that would be brought to justice in France in 1832; it would have been impossible to establish the identity of the parties who were to be in any way interested in these transactions.

It would be equally difficult to predict the duration of a single human life. Nevertheless, but few things are more determinate than the average duration of life in a multitude of individuals. One might well be amazed at the consonance of tabulated results, if he considers for a moment that the observations from which they were deduced were made at widely different times, and in countries which seemed to be under entirely different sanitary conditions.

The average duration of life in Rome, thirteen



hundred years ago, was the same as in Great Britain at the present time. Among the English nobles, "the expectation of life" at the age of eighty-four is about four years, and that of the fisherman at Ostend is precisely the same. With these results, the records of M. Deparcieux, made over a century ago, substantially agree. "The expectation," as deduced by Halley, from observations made in the seventeenth century, is but a little less, and is concordant with the "Combined Experience of Seventeen Companies," tabulated by Jenkin Jones in 1843. The Carlisle Table, computed by Mr. Milne from observations made in an English town by Dr. Heysham during the eight years prior to 1787, and the new American Table published in the Report of the Massachusetts Insurance Commissioner for 1868, are strikingly coincident.

The idea of ministering to the wants of mankind by making extended observations of births and deaths, and applying the theory of probabilities thereto, is of ancient origin. It did not in earlier times seem to meet with general appreciation. The older societies, ignoring very trustworthy data, evidently guessed at their rates of premium. They set sail like a ship without compass or chart, and in some instances were lucky enough to steer clear of the hidden reefs and to prosper on their voyage. The Amicable Society, which was organized in 1706, charged the same rates of premium on all ages between twelve and forty-five. This was said to be an entrance fee of £3 15s. per cent., and an annual premium of £5 per cent. Mr. Babbage endeavored, by examining the earlier records of this Society, to ascertain why £5 per cent. was taken, and concluded that it arose from the fact, that in the city of London the deaths were about one in twenty. At the present time it seems very strange that this

Society should have been content to grope in the dark, when the Breslau Table, published thirteen years previous to its organization, defined with a good degree of accuracy the probability of surviving at any age. "When," says Dr. Gouraud in "The History of the Calculus of Probabilities," "in 1693, an English mathematician of the highest order, proceeding in turn to study, in the obituary returns of London and Breslau, the general laws of human mortality, published on this subject a Memoir, which is even now read with admiration, absolutely no one took heed of it. \* \* \* \* \* Useless instructions! Buried in the vast and rich collection of Memoirs of the Royal Society of London, the admirable labors of Halley were only to be discovered by posterity."

Something like half a century after the organization of the Amicable Society, Thomas Simpson, a self-taught mathematician, after having established a reputation by the publication of various works on annuities, delivered a course of lectures in London, in which he excited the attention of the public by announcing the possibility of constructing a table of Assurance premiums, accurately graduated to the mortality of each age. Impelled by these statements, James Dodson turned his attention to the subject, and actually computed a table of rates. This circumstance, in combination with the increased diffusion of information on the subject of life contingencies, led, in 1762, to the establishment of the Equitable Society. This was the inauguration of a new era in the history of Assurance. Nothing that had previously existed was either safe or practicable. The new system, founded upon data obtained from actual observation, was established upon a scientific basis, and possessed the elements of stability.

This pencil of light stimulated further research.



The immense practical value of the subject was acknowledged, and to its consideration the attention of some of the ablest men in the kingdom was turned. Arrangements were made for more extended observation. The government aided in procuring the necessary statistical information. New tables were computed, each one verifying the financial safety of the results already attained. Extended comparisons were made between data obtained from various sources. Those tables found to be best graduated to the actual mortuary experience were accepted as standards, while others gradually grew obsolete. This process of comparing and eliminating has rendered the Rates of Mortality which are now in general use very trustworthy representatives of the decrement of human life.

A Life Assurance Society is founded on two assumptions, viz.: that the mortality among its members will be the same as that of the "Rate" which it has adopted as its basis; and that a certain percentage of interest can be realized on its investments. It is not a difficult, although a somewhat intricate, matter to compute upon these conditions a table of premiums which will make the business of Assurance practicable. A Company thus organized, if the original assumptions are just realized, is sure to be exactly solvent. It can pay every loss as it matures, and in paying the last will use every remaining dollar of its fund. If more favorable conditions are experienced, a surplus is realized; if less favorable, its bankruptcy is only a question of time. In order to accomplish its beneficent mission, the security of a Life Assurance Company should be as absolute as human forethought can make it. Hence, as a premium after a contract is made cannot be altered, the original assumptions should be such as will enable it to charge a larger

rate than will be required during the lifetime of the youngest member. What is not needed can be returned periodically to the assured. Practically, the selection of a rate of mortality, provided either of the standard tables are taken, is of less importance than the assumption of a per-centum of interest. The difference of one single unit in this element might, in time, either insure the prosperity of a Company or drive it into bankruptcy. By almost universal consent the rate adopted by the American Companies is four per cent. ; while the English Companies generally assume two and one-half or three per cent. To guard against adverse contingencies, and to provide for the expenses incident upon the conduct of business, it is customary to add to the net premium a margin or loading of from twenty to forty per cent. This constitutes what is usually known as the office premium.

In order to show the relation of the premium to the table of mortality, we will ascertain the net annual amount which would be paid to secure an assurance of one dollar, at the age of ninety years, assuming four per cent. interest, by the most intelligible arithmetical process by which the computation can be made.

By referring to the "Actuaries' Rate of Mortality," we find the number living and dying at the age of ninety, and at each subsequent year, is as follows :—

Age.	No. Living.	No. Dying.	Age.	No. Living.	No. Dying.
90	1319	427	95	89	52
91	892	322	96	37	24
92	570	231	97	13	9
93	339	155	98	4	3
94	184	95	99	1	1

The first step is to ascertain the present value of an annuity of one dollar at the age of ninety years, the first payment of which is to be made at once. This is equal to the payment for the first year, plus the present value of all future payments.

If thirteen hundred and nineteen persons were each to receive a payment of one dollar at the commencement of every year during the remainder of life, the first payment being made at once, is evidently equal to \$1319. At the commencement of the ninety-first year there would be but eight hundred and ninety-two of the thirteen hundred and nineteen persons alive; hence, but \$892 would be paid. If, at the commencement of this year, the Company were to make an adjustment for both years, it would pay, in addition to the \$1319 then due, a sum which, improved at the assumed rate of interest, would amount at the end of one year to \$892; or, in other words, its present worth. As every one of the annuitants has an equal claim upon this sum, it must be divided into thirteen hundred and nineteen parts. The share of each would be the present worth of  $\frac{892}{1319}$  of one dollar, or  $\frac{892}{1319}$  of unity, multiplied into the present value, one dollar, payable in one year. At the beginning of the third year there would be but five hundred and seventy of the thirteen hundred and nineteen living; consequently, but \$570 would be paid. Commuting this payment at the commencement of the ninetieth year, the share of each one would be  $\frac{570}{1319}$  of unity, multiplied into the present value of one dollar, payable two years hence. The present worth of all payments ascertained in this manner constitutes the value of a life annuity.

This process is completed in the following table:—

TABLE I.

*Age, . . . Ninety.*

Age.	Number Living.	The value of each one's share, unimproved.	Present worth of One Dollar.	Present worth of each Payment.
90	1319	$\frac{1319}{1319}$	$\times \$1.00000000$	$= \$1.00000000$
91	892	$\frac{892}{1319}$	$\times .96153846$	$= .6502536$
92	570	$\frac{570}{1319}$	$\times .92455625$	$= .3995429$
93	339	$\frac{339}{1319}$	$\times .88899636$	$= .2284885$
94	184	$\frac{184}{1319}$	$\times .85480419$	$= .1192448$
95	89	$\frac{89}{1319}$	$\times .82192711$	$= .0554598$
96	37	$\frac{37}{1319}$	$\times .79031353$	$= .0221696$
97	13	$\frac{13}{1319}$	$\times .75991781$	$= .0074897$
98	4	$\frac{4}{1319}$	$\times .73069020$	$= .0022168$
99	1	$\frac{1}{1319}$	$\times .70258674$	$= .0005326$
Present value of an annuity of One Dollar, \$2.485398 3				

The next step will be to ascertain the amount of the net single premium on a whole life policy of one dollar.

According to the Rate of Mortality, four hundred and twenty-seven, out of thirteen hundred and nineteen persons living at the age of ninety, will die within one year. If each one were assured for one dollar, and the losses adjusted at the end of the year, \$427 would then be due, and the share of each assurant entered at the commencement of the year would be  $\frac{1}{1319}$ th part of the \$427 or  $\frac{427}{1319}$  of one dollar. If the payment of the premium is made at the commencement of the year, such a sum must be taken as will, improved at the assumed rate of interest be equal to \$427 at the end of the year, or, in other words, its present value; and the share of each assurant will be  $\frac{427}{1319}$  of unity, multiplied by the present value of one dollar, payable one year hence. During the second year three hundred and twenty-two will die. If the whole number living at the commencement of the ninetieth year were assured for two years,



paying the assessment of both years at the outset, the whole amount due would be the present value of \$427, payable in one year, plus the present value of \$322, payable in two years. The share of each assurant would be  $\frac{427}{1319}$  of unity, multiplied into the present value of one dollar, payable in one year, added to  $\frac{322}{1319}$  of unity, multiplied into the value of one dollar, payable in two years. Adjusting in this manner, at the commencement of the ninetieth year, all losses for the remainder of life, the share of each one will be the net single premium. This is always equal to the present value of each individual's share of the amount paid for losses at the end of the first and of every subsequent year.

We have completed this process in the table below:

TABLE II.

*Age, . . . Ninety.*

Age.	Number Dying.	Assessment of each Assurant at the end of the year.	Present value of One Dollar.	Present value of each Year's Payment.
90	427	$\frac{427}{1319}$	× \$.96153846	= \$.3112788
91	322	$\frac{322}{1319}$	× .92455621	= .2257067
92	231	$\frac{231}{1319}$	× .88899636	= .1556922
93	155	$\frac{155}{1319}$	× .85480419	= .1004507
94	95	$\frac{95}{1319}$	× .82192711	= .0591868
95	52	$\frac{52}{1319}$	× .79031453	= .0311572
96	24	$\frac{24}{1319}$	× .75991781	= .0138272
97	9	$\frac{9}{1319}$	× .73069020	= .0049857
98	3	$\frac{3}{1319}$	× .70258674	= .0015980
99	1	$\frac{1}{1319}$	× .67556417	= .0005122
Net single premium for one dollar,				\$ .9043955

The present value of a life annuity of one dollar is equal to the present worth of one dollar per annum, to be received at the commencement of each year during life. The net annual premium is such an amount paid at the commencement of each



year, that the present value of the first and all subsequent premiums shall be equal to the net single premium. As the value of a life annuity is the present worth of a certain sum to be received at the commencement of each year, so the net single premium is the present worth of all the net annual premiums. Hence, the present value of an annuity of one dollar holds the same relation to one dollar, that the net single premium for one dollar holds to the net annual premium. Taking the value of an annuity of one dollar, and the net single premium for one dollar already obtained, and we have the following proportion :—

As the present value of the annuity is to one dollar, so is the net single premium to the net annual premium ; or,

$$\$2.4853983 : \$1 :: .9043955 : .36388312$$

\* The fourth term is the net annual premium for whole life assurance. It is, owing to loss of decimals, a little less than it should have been. The premium, more exactly, at this age is \$.36388844 ; consequently, the premium for one thousand dollars will be \$363.88844.

The correctness of this process is susceptible of proof. If we take at the end of each year from the fund made up from the premiums for the year, and the balance from preceding years, improved at four per cent., the amount of losses paid, we can meet every claim as it matures, and at the end of the ninety-ninth year, upon the payment of the last loss, the fund will be exhausted.

\* The method which we have employed in computing this premium lies at the foundation of all calculations involving premiums and annuities. There are extended abridgments by which the labor of computation is materially lightened, but our limits do not permit us to introduce them. We would refer the reader, for further information on the subject, to any standard work on annuities. "The Agent's Monetary and Life and Valuation Tables," by D. P. Fackler, contains in a very compact form the formulæ and tables in general use.

The following is a complete statement of the yearly account between the Company and thirteen hundred and nineteen members, assured for one thousand dollars each, by net annual premiums, entered at the age of ninety, upon the supposition that the assumptions upon which the premiums are based are just realized:—

## FIRST YEAR.

*Living, 1319; dying, 427.*

1319 premiums are.....	\$479,968.852
Improved at 4 per cent. interest, is.....	499,167.607
Deduct losses for the year.....	427,000.000
And the balance remaining is.....	72,167.607

## SECOND YEAR.

*Living, 892; dying, 322.*

The balance from the end of last year.....	\$ 72,167.607
And 892 premiums.....	324,588.488
Improved at interest, is.....	412,626.339
Deduct losses of.....	322,000.000
And balance remaining is.....	90,626.339

## THIRD YEAR.

*Living, 570; dying, 231.*

The balance from the end of last year.....	\$ 90,626.339
And 570 premiums.....	207,416.411
Improved at interest, is.....	309,964.459
Deduct losses of.....	231,000.000
And the balance remaining is.....	78,964.459

## FOURTH YEAR.

*Living, 339; dying, 155.*

The balance from the end of last year.....	\$ 78,964.459
And 339 premiums.....	123,358.181
Improved at interest, is.....	210,415.546
Deduct losses of.....	155,000.000
And the balance remaining is.....	55,415.546

## FIFTH YEAR.

*Living, 184; dying, 95.*

The balance from the end of last year.....	\$ 55,415.546
And 184 premiums.....	66,955.473
Improved at interest, is.....	127,265.859
Deduct losses of.....	95,000.000
And the balance remaining is.....	32,265.859

## SIXTH YEAR.

*Living, 89; dying, 52.*

The balance from the end of last year.....	\$ 32,265.859
And 89 premiums.....	32,386.071
Improved at interest, is.....	67,238.008
Deduct losses of .....	52,000.000
And the balance remaining is.....	15,238.008

## SEVENTH YEAR.

*Living, 37; dying, 24.*

The balance from the end of last year.....	\$ 15,238.008
And 37 premiums.....	13,463.872
Improved at interest, is.....	29,849.955
Deduct losses of.....	24,000.000
And the balance remaining is.....	5,849.955

## EIGHTH YEAR.

*Living, 13; dying, 9.*

The balance from the end of last year.....	\$ 5,849.955
And 13 premiums.....	4,730.550
Improved at interest, is.....	11,003.725
Deduct losses of.....	9,000.000
And balance remaining is.....	2,000.725

## NINTH YEAR.

*Living, 4; dying, 3.*

The balance from the end of last year.....	\$ 2,000.725
And 4 premiums.....	1,455.534
Improved at interest, is.....	3,597.650
Deduct losses of.....	3,000.000
And balance remaining is.....	597.650

## TENTH YEAR.

*Living, 1; dying, 1.*

The balance from the end of last year.....	\$ 597.650
And 1 premium.....	363.888
Improved at interest, is.....	1000.000
Deduct loss of.....	1000.000
And balance remaining is.....	000.000

The reader cannot fail to have observed that in the statement just given, the original assumptions in respect of mortality and interest are just realized, and that the Company is able to fulfill all its contracts, and no more. There is remaining at the end of every year, save the last, a considerable portion of the original premium unexpended, after providing for all losses. This constitutes what is usually called the reserve, or the cost of re-assurance. The portion of this fund belonging to an individual policy at the close of any year, is indicated by dividing the balance on hand, after paying all losses, by the number of assurers alive at the commencement of the next year. It would be well to notice the fact that in this example, while after the second year the whole amount of the reserve decreases, that of each policy, owing to the withdrawals by death, increases. At the end of the first year the reserve on a single policy was \$80.90; at the end of the fifth year it had increased to \$362.54, and to \$597.60 at the end of the ninth.

To illustrate more clearly the origin of the surplus, we would consider still further the assumptions upon which Companies are founded.

## I. Taking a case in which—

- 1st. The mortuary experiences of the Company exceeds the decrement of the assumed rate of mortality;
- 2d. The rate of interest realized is less than that assumed;



- 3d. The margin or loading is insufficient to provide for current expenses.

If the conclusions already reached are correct, it will need no argument to show the inherent unsoundness of a Company which is in the condition just described. The failure to realize the assumed rate of interest would alone leave the reserve inadequate, and the state of the Company would be rendered still worse by the encroachments upon the safety fund to meet losses and to provide for expenses. If it paid the earlier claims, it would inevitably fail to meet the later ones. By the accession of new members it might for years be able to meet its liabilities, but its ultimate failure would be as certain as the coming of seed-time and harvest.

II. Let us consider a case in which—

- 1st. The mortuary experience is less than the decrement of the assumed rate of mortality;
- 2d. The rate of interest actually realized is greater than that assumed;
- 3d. The margin or loading is more than sufficient to provide for current expenses.

Then a surplus would remain after providing the necessary reserves and meeting all claims for losses.

We are happy to be able to state that this is the case with a large majority of the American Companies. By very safe assumptions, prudent management, judicious investments, and careful medical selection of risks, their balance-sheets usually show, upon the periodic examination of their affairs, a considerable amount of surplus. The troublesome problem is,

HOW OUGHT THIS SURPLUS TO BE DIVIDED?

Various systems have been devised in answer to this interrogatory. Nearly every office has a plan



which, while it may bear a general resemblance to that of some other, is so modified as to be characteristic; and every Company, with not a little pertinacity, maintains that its system is fully as satisfactory as that of any of its neighbors. In accordance with their leading features, these plans may be divided as follows:—

- 1st. By a per-centage on the sums assured;
- 2d. By a per-centage on the sums assured, and on all reversionary additions thereto;
- 3d. By a per-centage on all premiums paid, and an additional per-centage on all previous dividends;
- 4th. By a per-centage upon all premiums paid;
- 5th. By reversionary additions to the policy;
- 6th. By the Contribution Plan.

The first and second of these systems are peculiar to the English Companies, and the sixth to American Companies.

The query would naturally be suggested as to which of these systems is the most desirable. The answer would be, that which establishes the most equitable relations between the Company and the individual policyholder.

Mr. Sang, an English actuary, while the business of Life Assurance in this country was in its infancy, very justly remarked:—

“Could the mortality amongst the members be accurately predicted, and the profits of investments foretold, the premiums could be computed so as just to meet the engagements; in which case there would never be any surplus fund; each member would receive exactly the benefit to which he is entitled; and, be it observed, no member can receive more than his share of the benefit, if not at the expense

of some other member who receives less. For security, somewhat more than the net value is charged as premium for assurance, and the excess of the actual over the net premium goes to form the profit fund. This excess, in fact, constitutes the profit on a policy."

These views indicate that what are usually termed profits, are simply the over-payments of the policyholders. The fund arises, as we have already seen, partly from the loading, partly from the excess of interest, and partly from a more favorable mortuary experience than that assumed. In equity this fund must be classed as a liability in which each policyholder has an interest. It is customary among business men to keep an exact account of all liabilities, and to pass to the credit of each of their customers whatever balance may be his due. Should a corporation endeavor to divide its profits by giving to each stockholder an equal amount, regardless of stock owned by each, it would soon find itself checked by a legal injunction, and, if necessary, its affairs would be placed in the hands of a receiver. A Mutual Life Assurance Company is a corporation in which the assurers are the stockholders, and their individual interest in the general fund varies with the size, age and class of their policies. From what source can a society of this kind derive any special rights or privileges in this matter not accorded to individuals or other corporations? Can it claim exemption from those usages which the common law has established between party and party? As it is as much bound to preserve equitable relations between itself and its various members as it is to maintain its own solvency, can it do any more or any less than to return to each one of the assured whatever of the surplus he may have contributed?

The system of division by which this is accomplished is known as the Contribution Plan.\*

To gain a clear comprehension of this plan, it will be necessary to ascertain the sources from which the surplus fund has been derived :—

1st. FROM INTEREST.—It is fair to presume that the funds paid by each individual policyholder have been improved in common with the funds paid by all the other policyholders; consequently, each one will be entitled to receive the same percentage of excess.

2d. FROM THE LOADING.—Of all matters connected with an Assurance society requiring accuracy of adjustment, the determination of the proportion of profit arising from this source due each assurant is, perhaps, the most difficult. The expenses during successive years is subject to considerable variation. Companies differ so materially in the details of their methods of transacting business, that it is impossible to give general directions.

3d. FROM THE INCREASED VITALITY.—The ratio of mortality given by the table is the basis from which the premium is computed. If the mortality experienced at any age was greater than that assumed, it would be just to charge the excess to the margin or loading of the premium at that age; if less, each policyholder should be credited with the proportion of the excess, and it should be returned to him at the periodic distribution.

\* In the Massachusetts Insurance Report for 1868, on page cxv., Mr. Sanford remarks :—"The Contribution Plan was first applied to the distribution of surplus by Mr. Sheppard Homans, the eminent actuary of the Mutual Life Insurance Company of New York, who has attributed to Mr. David Parks Fackler, his then assistant, now a consulting actuary of the same city, the suggestion of its idea, and has shared with him the credit of its discovery and development. The system was first applied to the distribution of the quinquennial dividend of this Company in 1863, since which time it has been adopted by many leading American Companies."

The most comprehensible method of adjustment between the Company and the assurants is an open debit and credit, in which the Company keeps the books. For the first, second and eleventh years the statement would be as follows:—

FIRST YEAR.

*Policy No. —.* *Cr.*

- 1st. By premium for the year.
- 2d. By interest at the rate actually realized from the Company's investments.

- Dr.* 1st. To cost of assurance of the year.
- 2d. To his share of the necessary expense.
- 3d. To the reserve held at the end of the year.
- 4th. To dividend to balance.

SECOND YEAR.

*Cr.*

- 1st. By reserve held at the end of the first year.
- 2d. By premium for the year.
- 3d. By interest on the sum of the premium and reserve.

- Dr.* 1st. To cost of assurance for the year.
- 2d. To expenses.
- 3d. To reserve at the end of the year.
- 4th. To dividend to balance.

ELEVENTH YEAR.

*Cr.*

(*On a Policy by Annual Payments.*)

- 1st. By reserve held at the end of the tenth year.
- 2d. By premium for the year.
- 3d. By interest.

(*On a Policy by Ten Annual Payments.*)

*Dr.*

- 1st. By reserve held at the end of the tenth year.
- 2d. By interest.

- 1st. To cost of assurance for the year.
- 2d. To expenses.
- 3d. To reserve at the end of the year.
- 4th. To dividend to balance.



On the credit side of the account we gave the statement for a policy by annual payments, and for a policy by ten annual payments. This was done to show the items passed to the credit of paid-up policies. The value of the elements used in this statement vary materially in different classes of policies, but the form is applicable to all.

It is evident that this subject is susceptible of an arithmetical elucidation. Taking, for example, a life policy for one thousand dollars, issued at the age of thirty-five, the payments on which are to be made by ten annual premiums, let us make the following assumptions, viz. :—

1st. That the premium is based upon the “Combined Experience,” or “Actuaries’ Rate of Mortality,” and loaded thirty per cent.;

2d. That the expenses shall be equivalent to ten per cent. of the premium, taken at the commencement of the year;

3d. That the mortality shall be two-thirds of the decrement of the table;

4th. That the Company shall realize seven per cent. interest on its investments.

Before proceeding further it will be necessary to ascertain the value of the various elements that enter into the account.

1st. *The Premium.*—We have already computed and shown the nature of the net annual premium. The premium on a ten-payment whole-life policy furnishes financially an exact equivalent to the Company for all the premiums on a whole-life policy by annual payments. It consists of two elements—the whole-life annual premium, and a sum which, improved at the rate of interest assumed in the computation of the tables, will purchase a deferred annuity equal to the whole-life annual premium, the



first payment of which is to be made at the commencement of the eleventh year.

A deferred annuity is one in which the payments are to commence at some future specified time. It is usually purchased either by a single premium, or by annual premiums, the last of which is to be made at the commencement of the year previous to that upon which the first payment of the annuity is due. To illustrate the process by which the amount of these premiums is determined, the present values of the yearly payments of the annuity computed in Table I. are here introduced.

Age.	Present Value of One Dollar.	Age.	Present Value of One Dollar.
90	1.0000000	95	.0554598
91	.6502596	96	.0221695
92	.3995429	97	.0074897
93	.2284835	98	.0022158
94	.1192448	99	.0005396

Present value of the first five payments.....\$2.3975308  
 Present value of the second five payments..... .0878674  
 Present value of the ten payments..... 2.4853982

The present value of a deferred annuity, entered at the age of ninety, the first payment of which is to be made at the commencement of the ninety-fifth year, is evidently the aggregate present values of one dollar to be received at the commencement of the ninety-fifth and of every subsequent year, or \$.0878674, the present value of the second five payments as indicated by the table. This is the net single premium. If paid by five annual premiums, the first of which is to be made at the beginning of the ninetieth year, such a sum must be taken annually as will, making due allowance for the chances of discontinuance by death, give a present value equal

to the single premium. In other words, the annual premium is equal to the yearly payment of a temporary annuity for five years, the value of which is \$.0878674. The present value of a temporary annuity of one dollar, as indicated by the table, for the ninetieth and the four subsequent years, is \$2.3975308. This amount holds the same relation to \$1 that \$.0878674, the single premium of a deferred annuity, does to the annual premium; or

$$\$2.3975308 : \$1 :: \$.0878674 : \$.03664449.$$

The net annual premium on a whole-life policy for one thousand dollars, issued at the age of thirty-five, is \$19.8665. The annual premium for a deferred annuity of \$19.8665, the first payment of which is to be made at the beginning of the eleventh year, at the age of forty-five, computed by the method just given, is \$22.1957. The sum (\$19.8665 + \$22.1957) of these two elements, or \$42.0622, constitutes the premium on a whole-life policy by ten annual payments. A margin or loading of thirty per cent. gives \$12.68 more, or an office premium of \$54.68. This is about the rate usually charged by first-class companies.

2d. *The Expenses* are, by our assumptions, to be equivalent to ten per cent. of the premium taken at the commencement of the year. This will leave \$49.21 as the effective office premium.

For convenience in calculation we shall consider this amount as the office rate, and make no charge for expenses. This allowance is intended not only to defray the agency and office charges during the time in which the premiums are paid, but also to provide for the creation of a fund to meet the expenses of subsequent years.

3d. *The Reserve*.—In the consideration of the statement of the account between the policyholders

and the Company made on page 17, it was remarked that the balance remaining at the end of each year, after the payment of all claims, was the reserve, and that the portion belonging to each policy could be readily ascertained by a simple division. This method of computation is not practicable. The net values of policies issued at different dates, and to persons of different ages, must be found separately. There are several methods employed for the valuation of life policies by annual payments. The following is as convenient a plan as any:—

The earlier assurant who has reached any given age must stand financially in the same relation to the Company as one who has been entered at that age, and the reserve retained must be such as will furnish an equivalent for the increase of the rate. The net single premium at the age of valuation is, as has been shown, equal to the value of an annuity of all the net annual premiums. The value of a policy at any time, together with the present value of all future premiums receivable, must be equal to the net single premium at the age of valuation. Hence the following rule:—

From the net single premium at the age of valuation, subtract the present value at that age of a life annuity of the net annual premium at the age of entry:

Net annual premium, age thirty-five.....	\$ 19.8665
Net single premium, age thirty-six.....	348.1712
Value of an annuity of \$1.00, age thirty-six.....	16.9475
Value of an annuity of \$19.8665, age thirty-six....	336.6875
Deduct from net single premium, and the balance	11.4837

is the reserve at the expiration of the first year, just before the second premium is due. Another plan of obtaining the same result is sometimes employed. It consists of finding the value of an annuity of the difference between the net premium at the age of entry, and that at the age of valuation:—

Net annual premium, age thirty-six.....	\$20.54404
Net annual premium, age thirty-five.....	19.86653
Difference.....	.67751
Value of an annuity of \$.67751, age thirty-six...	11.4837

The reserve on a ten-year policy is made up of two elements—the reserve on a simple life policy, and the sum of the accumulations of the payments of the deferred annuity. The accumulation of this payment the first year is the amount of the annuity, improved at the given rate of interest, divided by the probability of surviving\* during the year:—

Annual payment of deferred annuity of premium.	\$22.1957
Improved at four per cent. . . . .	23.0835
Divided by $\frac{81814}{82581}$ , the probability of surviving....	23.2999
Add reserve of policy by annual payments.....	11.4832
Reserve on a life policy by ten payments.....	34.7831

To obtain the reserve at the end of the second year, we add to the reserve of a whole-life policy the accumulations of both payments of the deferred annuity. At the end of the ten years, or at the age of forty-five, the reserve, the policy being paid up, will be just equal to the net single premium for whole-life assurance at the age of forty-five. The reserves during the remainder of life are, in every instance, just equal to the net single premium at the age of valuation.

This process can be verified as follows:—

If the original assumptions are realized, the amount remaining is the reserve. The amount actually assured each year is not the amount of the policy, but in every instance the amount of the policy less the reserve held at the end of the year. On the policy for \$1000 under consideration, the amount of the reserve at the end of the first year is \$34.78, and the amount really at risk is not \$1000, but \$1000 less \$34.78, or \$965.22. If at the end of the year we take the cost of assurance from the net premium, im-

\* For further explanation of this matter, see reprint of Mass. Ins. Co. Reports, page 366.



proved at four per cent., the balance will be identical with the reserve :—

Amount assured.....	\$965.220
Cost of assuring one dollar.....	.0092877
Whole cost of assurance.....	8.960
Net premium.....	42.062
Improved at four per cent.....	43.74
Less cost of assurance.....	34.78

The difficulty encountered in obtaining the reserve by this process is, that we cannot obtain the cost of assurance without the reserve. We can by considering \$1000 the amount insured, and using balance as the reserve, by successive repetitions of the process, reduce the error to an infinitesimal.

In Table III. the reserves on a ten-payment whole-life policy, issued at the age of thirty-five, are given. We also show the sum at risk each year by taking these reserves from the amount of the policy :—

TABLE III.

Age.	Amount of Policy.		Reserve at the end of the year.		Amount at Risk.
35	\$1,000	less	\$ 34.78	=	\$965.22
36	1,000	"	71.11	=	928.89
37	1,000	"	109.07	=	890.93
38	1,000	"	148.74	=	857.26
39	1,000	"	190.23	=	809.27
40	1,000	"	233.64	=	766.36
41	1,000	"	279.08	=	720.92
42	1,000	"	326.66	=	673.34
43	1,000	"	376.45	=	623.55
44	1,000	"	428.57	=	571.43
45	1,000	"	438.86	=	561.14
46	1,000	"	449.35	=	550.65
47	1,000	"	460.02	=	539.98
48	1,000	"	470.88	=	529.12
49	1,000	"	481.91	=	518.09
50	1,000	"	493.10	=	506.90



The reserve which the laws of Massachusetts compel every Company doing business in that State to maintain, is based on the same assumptions that have been taken in computing Table III. The policies are valued seriatim each year by the Commissioner, and the aggregate amount of the reserves in each Company is compared with its net assets. Its solvency is thus tested every twelve months.

4th. *The Cost of Assurance.*—By reference to the “Actuaries’ Rate of Mortality,” we find that the number living at the age of thirty-five is 85251, and that of this number 727 will die during the next year. The cost of assuring the monetary unit will be  $\frac{727}{85251}$  of that unit, or .0092877. This, multiplied by the number of units in any amount, will give at this age the cost of assuring that amount. By a similar method the cost of assuring any amount at any age can be readily ascertained. By our assumption the mortality was to be two-thirds of that of the table upon which the premium is based. The cost of assurance for the first year—the amount of the policy less the reserve, multiplied into two-thirds of cost of assuring one dollar—is given below:—

Amount of the policy is.....	\$1000.00
The reserve is.....	34.78
The amount at risk is.....	965.22
The tabular cost of assuring one dollar is.....	.0092877
Two-thirds of this cost is.....	.0061918
The cost of assuring amount at risk is.....	5.9764

In the table below this computation is given for the first sixteen years. The amount at risk is taken from Table III.

TABLE IV.

Age.	Amount at Risk.		Cost of Assur- ing Unity.		Tabular Cost of Assurance.		Two-thirds of Tabu- lar Cost.
35	\$965.22	×	.0092877	=	\$8.96	×	$\frac{2}{3}$ = \$5.98
36	928.89	×	.0094849	=	8.80	×	$\frac{2}{3}$ = 5.87
37	890.94	×	.0096867	=	8.62	×	$\frac{2}{3}$ = 5.75
38	851.26	×	.0099064	=	8.43	×	$\frac{2}{3}$ = 5.62
39	809.27	×	.0101311	=	8.18	×	$\frac{2}{3}$ = 5.46
40	766.36	×	.0103619	=	7.94	×	$\frac{2}{3}$ = 5.29
41	720.92	×	.0106118	=	7.65	×	$\frac{2}{3}$ = 5.10
42	673.34	×	.0108943	=	7.32	×	$\frac{2}{3}$ = 4.89
43	623.55	×	.0112509	=	7.02	×	$\frac{2}{3}$ = 4.67
44	571.43	×	.0116973	=	6.68	×	$\frac{2}{3}$ = 4.46
45	561.14	×	.0122120	=	6.85	×	$\frac{2}{3}$ = 4.57
46	550.65	×	.0128389	=	7.07	×	$\frac{2}{3}$ = 4.71
47	539.98	×	.0135157	=	7.29	×	$\frac{2}{3}$ = 4.86
48	529.12	×	.0142595	=	7.54	×	$\frac{2}{3}$ = 5.03
49	518.09	×	.0150611	=	7.80	×	$\frac{2}{3}$ = 5.20
50	506.90	×	.0159386	=	8.06	×	$\frac{2}{3}$ = 5.37

A margin has already been made for expenses by considering the office premium \$49.21, instead of \$54.68, the net premium loaded thirty per cent. Referring to Tables III. and IV. for the cost of assurance and the necessary reserves for each year, we will now lay before the reader a detailed statement of the account between the policy and the Company for the first sixteen years. The last item of debit in each year will be the amount of dividend due:—

## FIRST YEAR.

Age, . . . *Thirty-five.**Policy No. —*

	<i>Cr.</i>
By premium for the year.....	\$49.21
By interest at seven per cent.....	3.44
	<hr/>
	\$52.65
	<i>Dr.</i>
To cost of assurance for the year.....	\$ 5.98
To reserve at the end of the year.....	34.78
To dividend to balance.....	11.89
	<hr/>
	\$52.65

## SECOND YEAR.

*Age, . . . Thirty-six.**Cr.*

By reserve on hand at end of the first year.....	\$ 34.78
By premium for the year.....	49.21
By interest on premium and reserve.....	5.88
	<hr/>
	\$ 89.87

*Dr.*

To cost of assurance for the year.....	\$ 5.87
To reserve held at the end of the year.....	71.11
To dividend to balance.....	12.89
	<hr/>
	\$ 89.87

## THIRD YEAR.

*Age, . . . Thirty-seven.**Cr.*

By reserve at the end of the last year.....	\$ 71.11
By premium for the year.....	49.21
By interest.....	8.42
	<hr/>
	\$128.74

*Dr.*

To cost of assurance for the year.....	\$ 5.75
To reserve at the end of the year.....	109.06
To dividend to balance.....	13.93
	<hr/>
	\$128.74

## FOURTH YEAR.

*Age, . . . Thirty-eight.**Cr.*

By reserve held at the end of the last year.....	\$109.06
By premium for the year.....	49.21
By interest.....	11.08
	<hr/>
	\$169.35

*Dr.*

To cost of assurance for the year.....	\$ 5.62
To reserve at the end of the year.....	148.74
To dividend to balance.....	14.99
	<hr/>
	\$169.35

## FIFTH YEAR.

*Age, . . . Thirty-nine.**Cr.*

By reserve held at the end of the last year.....	\$148.74
By premium for the year.....	49.21
By interest.....	13.86
	<hr/>
	\$211.81

	<i>Dr.</i>
To cost of assurance for the year.....	\$ 5.46
To reserve at the end of the year.....	190.23
To dividend to balance.....	16.12
	<hr/>
	\$211.81

## SIXTH YEAR.

*Age, . . . . . Forty.*

	<i>Cr.</i>
By reserve held at the end of the last year.....	\$190.23
By premium for the year .....	49.21
By interest.....	16.76
	<hr/>
	\$256.25
	<i>Dr.</i>
To cost of assurance for the year.....	\$ 5.29
To reserve at the end of the year.....	233.64
To dividend to balance.....	17.32
	<hr/>
	\$256.25

## SEVENTH YEAR.

*Age, . . . . . Forty-one.*

	<i>Cr.</i>
By reserve held at the end of the last year.....	\$233.64
By premium for the year.....	49.21
By interest.....	19.80
	<hr/>
	\$302.65
	<i>Dr.</i>
To cost of assurance for the year.....	\$ 5.10
To reserve at the end of the year.....	279.08
To dividend to balance.....	18.47
	<hr/>
	\$302.65

## EIGHTH YEAR.

*Age, . . . . . Forty-two.*

	<i>Cr.</i>
By reserve held at the end of the last year .....	\$279.08
By premium for the year .....	49.21
By interest.....	22.98
	<hr/>
	\$351.27
	<i>Dr.</i>
To cost of assurance for the year.....	\$ 4.89
To reserve at the end of the year....	326.66
To dividend to balance.....	19.72
	<hr/>
	\$351.27

## NINTH YEAR.

*Age, . . . . . Forty-three.*

	<i>Cr.</i>
By reserve held at the end of the last year.....	\$326.66
By premium for the year.....	49.21
By interest.....	26.31
	<hr/>
	\$402.18



	<i>Dr.</i>
To cost of assurance for the year.....	\$ 4.67
To reserve for the year.....	376.45
To dividend to balance.....	21.06
	<hr/>
	\$402.18

## TENTH YEAR.

*Age, . . . Forty-four.*

	<i>Cr.</i>
By reserve held at the end of the last year.....	\$376.45
By premium for the year.....	49.21
By interest.....	29.79
	<hr/>
	\$455.45

	<i>Dr.</i>
To cost of assurance for the year.....	\$ 4.46
To reserve at the end of the year.....	428.57
To dividend to balance.....	22.42
	<hr/>
	\$455.45

## \*ELEVENTH YEAR.

*Age, . . . . Forty-five.*

	<i>Cr.</i>
By reserve held at the end of the last year.....	\$428.57
By interest.....	30.00
	<hr/>
	\$458.57

	<i>Dr.</i>
To cost of assurance for the year.....	\$ 4.57
To reserve at the end of the year.....	438.86
To dividend to balance.....	15.14
	<hr/>
	\$458.57

## TWELFTH YEAR.

*Age, . . . Forty-six.*

	<i>Cr.</i>
By reserve held at the end of the last year.....	\$438.86
By interest.....	30.71
	<hr/>
	\$469.57

	<i>Dr.</i>
To cost of assurance for the year.....	\$ 4.71
To reserve at the end of the year.....	449.35
To dividend to balance...	15.51
	<hr/>
	\$469.57

\* The premiums being now paid in full, we have on the credit side only the reserve from the end of the tenth year, improved at interest.

## THIRTEENTH YEAR.

*Age, . . . . . Forty-seven.**Cr.*

By reserve held at the end of the last year.....	\$449.35
By interest.....	31.47

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\$480.82*Dr.*

To cost of assurance for the year.....	\$ 4.86
To reserve at the end of the year.....	460.02
To dividend to balance.....	15.94

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\$480.82

## FOURTEENTH YEAR.

*Age, . . . . . Forty-eight.**Cr.*

By reserve held at the end of the last year.....	\$460.02
By interest.....	32.20

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\$492.22*Dr.*

To cost of assurance for the year.....	\$ 5.03
To reserve at the end of the year.....	470.88
To dividend to balance.....	16.31

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\$492.22

## FIFTEENTH YEAR.

*Age, . . . . . Forty-nine.**Cr.*

By reserve held at the end of the last year.....	\$470.88
By interest.....	32.96

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\$503.84*Dr.*

To cost of assurance for the year.....	\$ 5.20
To reserve at the end of the year.....	481.91
To dividend to balance.....	16.73

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\$503.84

## SIXTEENTH YEAR.

*Age, . . . . . Fifty.**Cr.*

By reserve held at the end of the last year.....	\$481.91
By interest.....	33.73

---

\$515.64*Dr.*

To cost of assurance for the year.....	\$ 5.37
To reserve at the end of the year.....	493.10
To dividend to balance.....	17.17

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\$515.64

These conditions are fully as favorable as have for any considerable length of time been experienced by any Company. It would be impossible, even if we knew the rate of interest that could be realized on investments, to predict the amount of future dividends, as the mortuary experience is very fluctuating and the expenses vary materially from year to year. Judging from past experience, it is probable that, taking the years together, the dividends in the six per cent. column of Table V. are fully as large as can be safely predicated for the future.

In order to show the effect of the realization of a rate of interest in excess of that used in the computation of premiums, we have given below a table in which a comparative statement is made between the dividends at seven per cent., as previously given, and five and six per cent., based upon the same assumptions in respect of mortality and loading.

TABLE V.

Year.	Four per cent.	Five per cent.	Six per cent.	Seven per cent.
1	\$10.42	\$10.91	\$11.40	\$11.89
2	10.37	11.21	12.05	12.89
3	10.32	11.53	12.73	13.93
4	10.24	11.82	13.34	14.99
5	10.17	12.16	14.14	16.12
6	10.09	12.48	14.88	17.32
7	9.98	12.81	15.62	18.47
8	9.87	13.15	16.54	19.72
9	9.78	13.54	17.30	21.06
10	9.65	13.91	17.96	22.42
11	2.28	6.67	10.94	15.14
12	2.36	6.74	11.11	15.51
13	2.43	6.94	11.43	15.94
14	2.53	7.11	11.71	16.31
15	2.60	7.36	12.02	16.73
16	2.70	7.54	12.35	17.17

It is a noteworthy fact that the surplus in the four per cent. column arises wholly from the loading and

mortuary experience assumed. Below is a table in which, with the same assumptions for loading and expenses as in the preceding table, we give dividends computed at four, five, six and seven per cent., upon the supposition that the mortuary experience is precisely that of the "rate" of mortality from which the premiums were calculated:—

TABLE VI.

Year.	Four per cent.	Five per cent.	Six per cent.	Seven per cent.
1	\$7.44	\$ 7.93	\$ 8.42	\$ 8.90
2	7.44	8.30	9.14	9.98
3	7.44	8.66	9.86	11.16
4	7.44	9.01	10.53	12.18
5	7.44	9.44	11.42	13.40
6	7.44	9.83	12.23	14.67
7	7.44	10.26	13.07	15.92
8	7.44	10.71	14.10	17.28
9	7.44	11.21	14.97	18.73
10	7.44	11.69	15.94	20.20
11	0.00	4.29	8.57	12.86
12	0.00	4.39	8.78	13.17
13	0.00	4.49	8.99	13.48
14	0.00	4.60	9.20	13.80
15	0.00	4.71	9.42	14.13
16	0.00	4.82	9.64	14.46

In the four per cent. column the dividends arise wholly from the loading. In the five, six and seven per cent. columns it comes from both loading and interest.

An analysis of the dividends of any year will show the portion that has been contributed by the margin, the excess of interest, and the increased vitality of the assurers. By way of illustration, we will take the dividend of the second year, given in the seven per cent. column of Table V, the amount of which was \$12.89:—



## CONTRIBUTION FROM THE MARGIN.

The office premium was.....	\$49.21
The net premium was.....	42.06
The margin was.....	7.15
Improved at seven per cent. is the contribution.....	7.65

## CONTRIBUTION FROM INTEREST.

The reserve was.....	\$34.78
The net premium was.....	42.06
The amount was.....	76.84
The amount of interest received at seven per cent. was	5.38
Amount required by our assumption at four per cent. was	3.07
Contribution.....	2.31

## CONTRIBUTION FROM VITALITY.

Tabular cost of assurance was.....	\$ 8.80
Actual cost was.....	5.87
Contribution.....	2.93

## RECAPITULATION.

Contribution from the margin.....	\$ 7.65
Contribution from interest.....	2.31
Contribution from increased vitality.....	2.93
Whole amount of contribution.....	12.89

In making up the interest account, it will be observed that we took the sum of the reserve and the net premium as the fund, as we had already improved the loading at the rate of interest actually received; to have brought it into the general fund would have given us a twofold gain from its interest.

An objection has been urged against the Contribution Plan on account of its complexity, and the amount of labor involved in making the necessary computations. This objection inures with equal force against all corporations that conduct an extended business. Indeed, the amount of labor required is not as great as might be supposed, as, from the peculiar organization of an Assurance society, it admits of extended abridgments, of which ordinary business transactions are not susceptible.

It will be noticed that the dividends are adjusted to the actual mortuary experience of the Company. In order to ascertain what this experience is, it will be necessary to classify all members according to their ages, regardless of the time at which they were assured. This will enable the Company to assess the premiums paid by the individual members of any class to meet the losses that have occurred in that class between the periodic distributions. This implies a breadth of base which exists only in the larger Companies; in smaller Companies the mortality will not be sufficiently uniform, particularly upon the extreme ages, to enable the Company to proceed in this manner. One of two alternatives is then left, viz.: to ascertain the per-centage of mortality on all ages taken together in comparison with the table; or to take an average per cent. of the past mortuary experience of the Company upon any given age, and to increase or diminish this per-centage as the general mortality for the given year upon all ages is greater or less.

Having ascertained the mortality upon the different ages, and the per-centum of assessment, we can proceed with another system of classification. All policies of the same sort—that is, policies issued during the same fiscal year, upon the same plan, and to persons of the same age—may be grouped together and treated as one large policy, representing the aggregate amount of all the smaller ones. The dividend on this large policy can be ascertained, and each of the smaller policies credited with its proportion. By this process the work of computation can be abridged, as we need have only as many accounts as we have different sorts of policies.

The reader cannot fail to have observed that the dividends, with the exception of the four per cent. column in Table IV., uniformly increase till the end

of the tenth year; that they decrease at the end of the eleventh year, and then constantly increase. This increase continues till the policy becomes a claim. The decrease at the end of the eleventh year is owing to the fact that, there being no payments after the tenth year, during that and the succeeding years no surplus arises from the margin or loading of the premium. The dividends succeeding the tenth arise, in Table V., from the excess of interest and from very favorable mortuary experience, and, in Table VI., from excess of interest alone.

The facts developed in the tables show the inequity of the uniform per-centage plan of distributing surplus. The younger members would receive more than was their due, at the expense of the older ones. Practically, not only is this fact true, but the policies on one plan take part of the fund earned by policies on another, and the assured at one age the over-payments of those of another; for the loading of the premiums on different plans is not always the same, and the mortality of different ages is often very far from holding the same relative correspondence to the table from which the premiums were computed.

It is customary to hold at the end of the year, in addition to the reserve, a small fund to guard against adverse contingencies. This, in Companies making an annual distribution to all policies in force at the end of the year, is usually about four per cent.

The system of distribution that we have given is very nearly equitable, but not exactly so. At the outset none but healthy lives are accepted, but, once taken, the Company is bound by the conditions of the contract to retain them. The physical condition of the assured gradually deteriorates, not only from the inroads of disease, but from the fact that, as a rule, only the best risks withdraw. Thus the

Company, no matter how carefully it makes its selection at the outset, in time has a large number of impaired risks on its hands. The mortality among these is greater than among those who have recently been assured. Grouping the old and new members together, and assessing all equally to pay the losses, would bear too heavily upon the new ones. This is a matter of secondary importance, but still it is worthy of remark.

The influence of occupation upon the duration of life is a more noteworthy matter, which, while it has an important bearing upon the subject of assurance, is at present imperfectly understood. Nothing is more certain than that some employments tend to prolong life, while others shorten it. To a certain extent, this is a recognized element in the determination of the cost of assurance. Extra rates are charged to those who are engaged in hazardous employments, but no abatement is made to those who are surrounded by the most favorable sanitary conditions. It is difficult, from the want of proper data, to determine with any degree of accuracy to what extent this should affect the cost of assurance. While Companies substantially agree in their premiums for healthy assurants, engaged in the ordinary vocations of life, their extra rates for hazardous risks vary materially. With increased diffusion of light, the practice of Companies will undoubtedly be modified.

Life Assurance Societies are an outgrowth of modern civilization. Their aid is invoked to avert the financial calamity incident upon the failure of a productive life. By depositing annually a small sum, one can not only make provision for his declining years, but, in the event of premature death, for those who are dependent upon him. By no other system can this be accomplished.



The vast accumulations held by these institutions are trust funds for the widow and the fatherless. The little deposits, coming back at a time when they are most needed, affording sustenance to childhood and support to old age, seem almost to be freighted with a double blessing.

With so great an outreach into the future, common prudence would seem to suggest that all the guards which human foresight could devise should be thrown around these institutions. The best scientific and financial talent should be called to the management of their affairs. Their security should be beyond question. Solvency first, and then equity, should be their watchwords. It is only by a high sense of the responsibility resting upon them, and by fidelity to the high trusts committed to them, that these institutions can secure the accomplishment of the benign mission which it is their purpose to fulfill.

TABLE I.

*Net Rates—"Actuaries'" Four Per Cent. Annuities and Single Premiums.*

Present Value of \$1.00 per Annum, to be received at the commencement of every Year during Life.				Single Premium for whole Life Assurance for One Thousand Dollars.			
Age.		Age.		Age.		Age.	
10	\$20.454	43	\$15.374	10	\$213.32	43	\$403.71
11	20.369	44	15.119	11	216.56	44	418.52
12	20.282	45	14.857	12	219.93	45	428.57
13	20.191	46	14.590	13	223.44	46	438.86
14	20.096	47	14.317	14	227.08	47	449.35
15	19.998	48	14.039	15	230.86	48	460.02
16	19.896	49	13.757	16	234.78	49	470.88
17	19.790	50	13.470	17	238.84	50	481.91
18	19.681	51	13.179	18	243.05	51	493.11
19	19.567	52	12.884	19	247.40	52	504.46
20	19.450	53	12.585	20	251.91	53	515.95
21	19.330	54	12.283	21	256.56	54	527.57
22	19.204	55	11.978	22	261.38	55	539.31
23	19.075	56	11.670	23	266.36	56	551.16
24	18.941	57	11.359	24	271.50	57	563.10
25	18.803	58	11.046	25	276.82	58	575.14
26	18.660	59	10.731	26	282.31	59	587.26
27	18.512	60	10.415	27	287.99	60	599.43
28	18.360	61	10.098	28	293.86	61	611.63
29	18.202	62	9.780	29	299.91	62	623.83
30	18.040	63	9.464	30	306.17	63	636.00
31	17.872	64	9.149	31	312.62	64	648.12
32	17.698	65	8.835	32	319.29	65	660.17
33	17.520	66	8.525	33	326.17	66	672.13
34	17.335	67	8.217	34	333.27	67	683.96
35	17.144	68	7.913	35	340.60	68	695.66
36	16.948	69	7.613	36	348.17	69	707.19
37	16.744	70	7.317	37	355.99	70	718.57
38	16.534	71	7.026	38	364.07	71	729.76
39	16.317	72	6.740	39	372.42	72	740.76
40	16.093	73	6.459	40	381.04	73	751.57
41	15.861	74	6.184	41	389.96	74	762.15
42	15.621	75	5.915	42	399.18	75	772.51

TABLE II.

*Net Rates—"Actuaries'" Four Per Cent. Premiums for a Whole Life Assurance of One Thousand Dollars.*

By ANNUAL PAYMENTS.				• By TEN ANNUAL PAYMENTS.			
Age.		Age.		Age.		Age.	
10	10.43	43	26.58	10	26.02	43	51.08
11	10.63	44	27.68	11	26.42	44	52.44
12	10.84	45	28.84	12	26.83	45	53.86
13	11.07	46	30.08	13	27.27	46	55.33
14	11.30	47	31.38	14	27.72	47	56.85
15	11.54	48	32.77	15	28.19	48	58.43
16	11.80	49	34.23	16	28.68	49	60.05
17	12.07	50	35.78	17	29.18	50	61.74
18	12.35	51	37.42	18	29.71	51	63.49
19	12.64	52	39.15	19	30.25	52	65.30
20	12.95	53	40.97	20	30.81	53	67.17
21	13.27	54	42.95	21	31.40	54	69.12
22	13.61	55	45.03	22	32.00	55	71.14
23	13.96	56	47.23	23	32.63	56	73.25
24	14.33	57	49.57	24	33.27	57	75.44
25	14.72	58	52.07	25	33.94	58	77.74
26	15.13	59	54.72	26	34.64	59	80.15
27	15.58	60	57.56	27	35.35	60	82.68
28	16.00	61	60.57	28	36.09	61	85.34
29	16.48	62	63.78	29	36.86	62	88.13
30	16.97	63	67.20	30	37.66	63	91.07
31	17.49	64	70.84	31	38.48	64	94.16
32	18.04	65	74.72	32	39.33	65	97.43
33	18.62	66	78.85	33	40.21	66	100.88
34	19.23	67	83.24	34	41.12	67	104.53
35	19.89	68	87.91	35	42.06	68	108.39
36	20.54	69	92.89	36	43.04	69	112.48
37	21.26	70	98.20	37	44.05	70	116.85
38	22.02	71	103.87	38	45.10	71	121.50
39	22.82	72	109.91	39	46.20	72	126.48
40	23.68	73	116.36	40	47.34	73	131.81
41	24.59	74	123.25	41	48.53	74	137.53
42	25.55	75	130.48	42	49.77	75	143.68





TABLE I.

## COMPOUND INTEREST,

*Showing the amount of \$1, improved at Compound Interest, for any number of years not exceeding 100.*

Years.	4 per Ct.	4½ per Ct.	5 per Ct.	6 per Ct.	7 per Ct.	8 per Ct.
1	1.040000	1.045000	1.050000	1.060000	1.070000	1.080000
2	1.081600	1.092025	1.102500	1.123600	1.144900	1.166400
3	1.124864	1.141166	1.157625	1.191016	1.225043	1.259712
4	1.169859	1.192519	1.215506	1.262477	1.310796	1.360489
5	1.216653	1.246182	1.276282	1.338226	1.402552	1.469328
6	1.265319	1.302260	1.340096	1.418519	1.500730	1.586874
7	1.315932	1.360862	1.407100	1.503630	1.605781	1.713824
8	1.368569	1.422101	1.477455	1.593848	1.718186	1.850930
9	1.423312	1.486095	1.551328	1.689479	1.838459	1.999005
10	1.480244	1.552969	1.628895	1.790848	1.967151	2.158925
11	1.539454	1.622853	1.710339	1.898299	2.104852	2.331639
12	1.601032	1.695881	1.795856	2.012196	2.252192	2.518170
13	1.665074	1.772196	1.885649	2.132928	2.409845	2.719624
14	1.731676	1.851945	1.979932	2.260904	2.578534	2.937194
15	1.800944	1.935282	2.078928	2.396558	2.759032	3.172169
16	1.872931	2.022370	2.182875	2.540352	2.952164	3.425943
17	1.947901	2.113377	2.292018	2.692773	3.158815	3.700018
18	2.025817	2.208479	2.406619	2.854339	3.379932	3.996020
19	2.106849	2.307860	2.526950	3.025600	3.616528	4.315701
20	2.191123	2.414714	2.653298	3.207135	3.869684	4.660957
21	2.278768	2.520241	2.785963	3.399564	4.140562	5.033834
22	2.369919	2.633652	2.925261	3.603537	4.430402	5.436540
23	2.464716	2.752166	3.071524	3.819750	4.740530	5.871464
24	2.563304	2.876014	3.225100	4.048935	5.072367	6.341181
25	2.665836	3.005434	3.386355	4.291871	5.427433	6.848475
26	2.772470	3.140679	3.555673	4.549383	5.807353	7.396353
27	2.883369	3.282010	3.733456	4.822346	6.213868	7.988061
28	2.998703	3.429700	3.920129	5.111687	6.648838	8.627106
29	3.118651	3.584036	4.116136	5.418388	7.114257	9.317275
30	3.243398	3.745318	4.321942	5.743491	7.612255	10.062657
31	3.373133	3.913857	4.538039	6.088101	8.145113	10.867669
32	3.508039	4.089931	4.764941	6.453387	8.715271	11.737083
33	3.648381	4.274030	5.003189	6.840590	9.325340	12.676050
34	3.794316	4.466362	5.253348	7.251025	9.978114	13.690134
35	3.946089	4.667348	5.516015	7.686087	10.676581	14.785344
36	4.103933	4.877778	5.791816	8.147252	11.423942	15.968172
37	4.268090	5.096460	6.081407	8.636087	12.223618	17.245626
38	4.438336	5.326219	6.385477	9.154232	13.079271	18.625276
39	4.616366	5.566499	6.704751	9.703507	13.994820	20.115298
40	4.801021	5.816365	7.039939	10.285718	14.974458	21.724522
41	4.993061	6.078101	7.391988	10.902861	16.022670	23.462483
42	5.192784	6.351615	7.761588	11.557033	17.144257	25.339482
43	5.400495	6.637438	8.149667	12.254553	18.344355	27.366640
44	5.616515	6.936123	8.557110	12.985482	19.628460	29.559712
45	5.841176	7.248248	8.995088	13.764611	21.002452	31.920449
46	6.074823	7.574420	9.434258	14.590487	22.472623	34.474085
47	6.317816	7.915268	9.905971	15.465917	24.045717	37.222012
48	6.570528	8.271456	10.401270	16.398722	25.728907	40.210573
49	6.833349	8.643671	10.921333	17.377104	27.529930	43.427419
50	7.106683	9.032636	11.467400	18.420154	29.457025	46.901613

TABLE II.  
COMPOUND INTEREST.

*The amount of \$1 per annum in any number of Years.*

Years.	4 per Cent.	5 per Cent.	6 per Cent.	7 per Cent.	8 per Cent.
1	1.000000	1.000000	1.000000	1.000000	1.000000
2	2.040000	2.050000	2.060000	2.070000	2.080000
3	3.121600	3.152500	3.183600	3.214900	3.246400
4	4.246464	4.310125	4.374616	4.439943	4.506112
5	5.416323	5.525631	5.637093	5.750739	5.866601
6	6.632975	6.801913	6.975319	7.155291	7.335929
7	7.898294	8.147008	8.393833	8.654621	8.922803
8	9.214226	9.549109	9.897468	10.259803	10.636628
9	10.582795	11.026564	11.491316	11.977989	12.487558
10	12.006107	12.577893	13.180795	13.816448	14.486562
11	13.486351	14.206787	14.971643	15.783599	16.645487
12	15.025805	15.917127	16.869941	17.888451	18.977126
13	16.626838	17.712983	18.882138	20.140643	21.495297
14	18.291911	19.598632	21.015066	22.550488	24.214920
15	20.023588	21.578564	23.275970	25.129022	27.152114
16	21.824531	23.657492	25.672528	27.888054	30.324283
17	23.697512	25.840366	28.212880	30.840217	33.750226
18	25.645413	28.132385	30.905653	33.999033	37.450244
19	27.671229	30.539004	33.759992	37.378965	41.446263
20	29.778079	33.065954	36.785591	40.995492	45.761964
21	31.969202	35.719252	39.992727	44.865177	50.422921
22	34.247970	38.505214	43.392290	49.005739	55.456755
23	36.617889	41.430475	46.995828	53.436141	60.893296
24	39.082604	44.501999	50.815577	58.176671	66.764759
25	41.645908	47.727099	54.864512	63.249038	73.105940
26	44.311745	51.113454	59.156383	68.676470	79.954415
27	47.084214	54.669126	63.705766	74.483823	87.350763
28	49.967583	58.402583	68.528112	80.697691	95.338830
29	52.966286	62.322712	73.639798	87.346529	103.965936
30	56.084938	66.438848	79.058186	94.460786	113.283211
31	59.328335	70.760790	84.801677	102.073041	123.345868
32	62.701469	75.298829	90.889778	110.218154	134.213537
33	66.209527	80.063771	97.343165	118.933425	145.950620
34	69.857909	85.066959	104.183755	128.258765	158.626670
35	73.652225	90.320307	111.434780	138.236678	172.316804
36	77.583314	95.836323	119.120867	148.913460	187.102148
37	81.702246	101.628139	127.268119	160.337402	203.070820
38	85.970336	107.709546	135.904206	172.561020	220.315945
39	90.409150	114.095023	145.058458	185.640292	238.941221
40	95.025516	120.799774	154.761966	199.635112	259.056519
41	99.826536	127.839763	165.047684	214.609570	280.781040
42	104.819598	135.231751	175.950545	230.632240	304.243523
43	110.012382	142.993339	187.507577	247.776496	329.583005
44	115.412877	151.143006	199.758032	266.120851	356.949646
45	121.029392	159.700156	212.743514	285.749311	386.505617
46	126.870568	168.685164	226.508125	306.751763	418.426067
47	132.945390	178.119422	241.098612	329.224386	452.900152
48	139.263206	188.025393	256.564529	353.270093	490.132164
49	145.833734	199.426663	272.958401	378.999000	530.342737
50	152.667084	209.347996	290.335905	406.528929	573.770156

TABLE III.

## THE PRESENT VALUE OF ONE DOLLAR PER ANNUM,

*For any number of years to 50, at 4, 5, 6, 7 and 8 per cent. interest.*

Years.	4 per Cent.	5 per Cent.	6 per Cent.	7 per Cent.	8 per Cent.
1	.961538	.952381	.943396	.934579	.925926
2	1.886095	1.859410	1.833393	1.808018	1.783265
3	2.775091	2.723248	2.673012	2.624316	2.577097
4	3.629895	3.545951	3.465106	3.387211	3.312127
5	4.451822	4.329477	4.212364	4.100197	3.992710
6	5.242137	5.075692	4.917324	4.766540	4.622880
7	6.002055	5.786373	5.582381	5.389289	5.206370
8	6.732745	6.463213	6.209794	5.971299	5.746639
9	7.435332	7.107822	6.801692	6.515232	6.246888
10	8.110896	7.721735	7.360087	7.023582	6.710081
11	8.760477	8.306414	7.886875	7.498674	7.133964
12	9.386074	8.863252	8.383844	7.942686	7.536078
13	9.985648	9.393573	8.852683	8.357651	7.903776
14	10.563123	9.898641	9.294984	8.745468	8.244237
15	11.118387	10.379658	9.712249	9.107914	8.559479
16	11.652296	10.837770	10.105895	9.446649	8.851369
17	12.165669	11.274086	10.477260	9.763223	9.121633
18	12.659297	11.689587	10.827603	10.059687	9.371887
19	13.133939	12.085321	11.158116	10.335595	9.603599
20	13.590326	12.462210	11.469921	10.594014	9.818147
21	14.029160	12.821153	11.764077	10.835527	10.016803
22	14.451115	13.163003	12.041582	11.061241	10.200744
23	14.856842	13.488574	12.303379	11.272187	10.371059
24	15.246963	13.798642	12.550358	11.469334	10.528758
25	15.622080	14.093345	12.783356	11.653583	10.674776
26	15.982769	14.375185	13.003166	11.825779	10.809978
27	16.329586	14.643034	13.210534	11.986709	10.935165
28	16.663063	14.898127	13.406164	12.137111	11.051078
29	16.983715	15.141074	13.590721	12.277674	11.158406
30	17.292033	15.372451	13.764831	12.409041	11.257783
31	17.588494	15.592811	13.929086	12.531814	11.349799
32	17.873552	15.802677	14.084043	12.646555	11.434999
33	18.147646	16.002549	14.230230	12.753790	11.513888
34	18.411193	16.192904	14.368141	12.854009	11.516934
35	18.664613	16.374194	14.498246	12.947672	11.654568
36	18.908282	16.546852	14.620937	13.035208	11.717193
37	19.142579	16.711287	14.736780	13.117017	11.775179
38	19.367864	16.867893	14.846019	13.193473	11.828869
39	19.584485	17.017011	14.949075	13.264928	11.878582
40	19.792774	17.159086	15.046297	13.331709	11.924613
41	19.993052	17.294368	15.138016	13.394120	11.967235
42	20.185627	17.423208	15.224543	13.452449	12.006699
43	20.370795	17.545912	15.306173	13.506962	12.043240
44	20.548841	17.662773	15.383182	13.557908	12.077074
45	20.720040	17.774070	15.455832	13.605522	12.108402
46	20.884654	17.880067	15.524370	13.650020	12.137409
47	21.042336	17.981916	15.589928	13.691603	12.164267
48	21.195131	18.077158	15.650027	13.730474	12.189136
49	21.341472	18.168722	15.707572	13.766799	12.212163
50	21.482185	18.255925	15.761861	13.800746	12.233485



TABLE IV.

## THE PRESENT VALUE OF ONE DOLLAR,

*Due at the end of any number of years to 100, at 4, 5, 6, 7 and 8 per cent. interest.*

Years.	4 per Cent.	5 per Cent.	6 per Cent.	7 per Cent.	8 per Cent.
1	.96153846	.95238095	.94339623	.93459744	.92592593
2	.92455621	.90702343	.88999644	.87343873	.85733882
3	.88899636	.86383760	.83961928	.81629788	.79383224
4	.85480419	.82276247	.79209366	.76289521	.73502985
5	.82192711	.78352616	.74725817	.71298618	.68058320
6	.79031453	.74621540	.70496054	.66634222	.63016963
7	.75991781	.71068133	.66505711	.62274974	.58349040
8	.73069020	.67683936	.62741237	.58200910	.54026888
9	.70258674	.64460892	.59189846	.54393374	.50024897
10	.67556417	.61391325	.55839478	.50834929	.46319349
11	.64958093	.58467929	.52678753	.47509280	.42888286
12	.62459705	.55683742	.49696936	.44401196	.39711376
13	.60057409	.53032135	.46883902	.41496445	.36769792
14	.57747508	.50506795	.44230096	.38781724	.34046104
15	.55526450	.48101710	.41726506	.36244602	.31524171
16	.53390818	.45811152	.39364628	.33873460	.29189047
17	.51337325	.43629669	.37136442	.31657439	.27026895
18	.49362812	.41552065	.35034379	.29586392	.25024903
19	.47464242	.39573396	.33051301	.27650833	.23171206
20	.45638695	.37688948	.31180473	.25841900	.21454821
21	.43883360	.35894236	.29415540	.24151309	.19865575
22	.42195539	.34184987	.27750510	.22571317	.18394051
23	.40572633	.32557131	.26179726	.21094688	.17031528
24	.39012147	.31006791	.24697855	.19714662	.15769934
25	.37511680	.29530277	.23299863	.18424918	.14601790
26	.36068923	.28124073	.21981003	.17219549	.13520176
27	.34681657	.26784832	.20736795	.16093037	.12518682
28	.33347747	.25509364	.19563014	.15040221	.11591372
29	.32065141	.24294632	.18455674	.14056282	.10732752
30	.30831867	.23137745	.17411013	.13136712	.09937733
31	.29646026	.22035947	.16425484	.12277301	.09201605
32	.28505794	.20986617	.15495740	.11474113	.08520005
33	.27409417	.19987254	.14618622	.10723470	.07888893
34	.26355209	.19035480	.13791153	.10021934	.07304531
35	.25341547	.18129029	.13010522	.09366294	.06763454
36	.24366872	.17265741	.12274077	.08753546	.06262458
37	.23429635	.16443563	.11579318	.08180884	.05798572
38	.22528543	.15660536	.10923885	.07645686	.05369048
39	.2166261	.14914797	.10305552	.07145501	.04971341
40	.20828904	.14204568	.09722219	.06678038	.04603093
41	.20027792	.13523130	.09171905	.06241157	.04262123
42	.19257493	.12883962	.08652740	.05832857	.03946411
43	.18516820	.12270440	.08162982	.05451268	.03654084
44	.17804635	.11686133	.07700908	.05094643	.03383411
45	.17119841	.11129651	.07265007	.04761349	.03132788
46	.16461386	.10599668	.06853781	.04449859	.02900730
47	.1582256	.10094921	.06465831	.04158747	.02685861
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49	.14634112	.09156391	.05754566	.03632410	.02302693
50	.14071262	.08720373	.05428836	.03394776	.02132123





